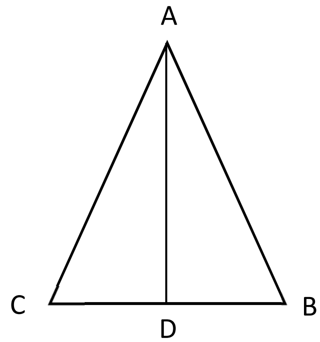
Math 213 Exam 2A Fall 2018

Name____

1. (6 pts) For each of the following, identify what information needs to be added to the given so it is possible to justify the conclusion.



a. Given: $\overline{AD}$ *bisects* $< CAB$

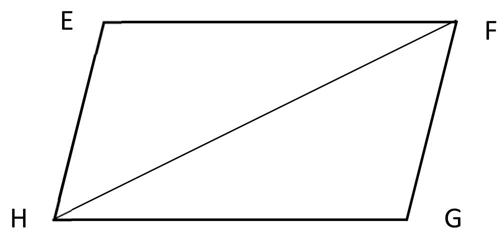
Prove: $\Delta ACD \cong \Delta ABD$

We need either

$\overline{AC} \cong \overline{AB}$ (SAS)

or $\angle ACD \cong \angle ABD$ (AAS)

or $\angle ADC \cong \angle ADB$ (AAS)



b. Given: $< EFH \cong < GHF$

Prove: EFGH is a parallelogram

We need either

$\overline{EF} \cong \overline{HG}$ (SAS)

or $\angle HEF \cong \angle HGF$ (AAS)

or $\angle EHF \cong \angle HFG$ (AAS)

2. (10 pts) Consider the following statement: **The diagonals of an isosceles trapezoid are congruent.**

Circle the letter by the statements below which are **equivalent** to the given statement in bold.

(a.) If a trapezoid is isosceles then its diagonals are congruent. ($p \rightarrow q$)

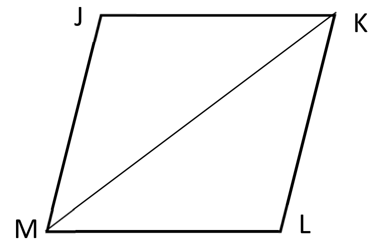
b. If a trapezoid has congruent diagonals then it is isosceles. $q \rightarrow p$

(c.) All isosceles trapezoids have congruent diagonals. $p \rightarrow q$

d. Only trapezoids with congruent diagonals are isosceles. $p \leftrightarrow q$

(e.) If a trapezoid does not have congruent diagonals then it is not isosceles. $\sim q \rightarrow \sim p$

3. (16 pts) Fill in the missing steps in the proof below.



Given: JKLM is a parallelogram

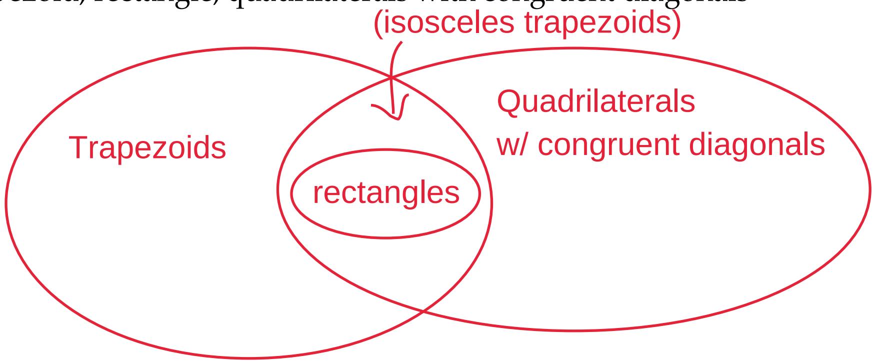
$\overline{KM}$ bisects $< JKL$ and $< JML$

Prove: JKLM is a rhombus

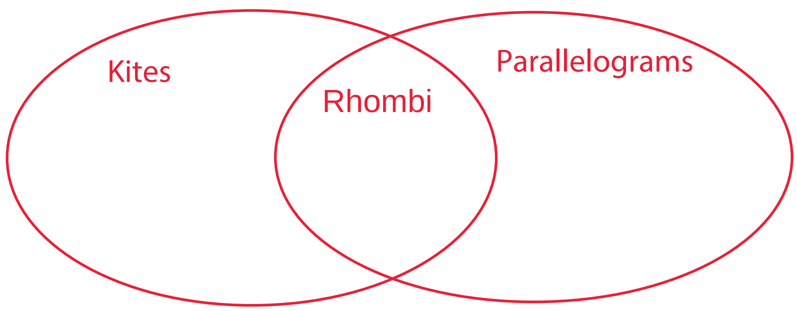
Statement	Reason
1. JKLM is a parallelogram $\overline{KM}$ bisects $< JKL$ and $< JML$	1. Given
2. $< JKM \cong < LKM$	2. Def'n of angle bisector
3. $< JMK \cong < LMK$	3. Def'n of angle bisector
4. $\overline{KM} \cong \overline{KM}$	4. Reflexive property
5. $\Delta JKM \cong \Delta LKM$	5. ASA
6. $\overline{JK} \cong \overline{KL}$	6. CPCTC
7. $\overline{JK} \cong \overline{ML}$ and $\overline{JM} \cong \overline{KL}$	7. Opposite sides of a parallelogram are congruent
8. $\overline{JK} \cong \overline{ML} \cong \overline{JM} \cong \overline{KL}$	8. Substitution
9. JKLM is a rhombus	9. Def'n of rhombus

4. (12 pts) Make a Venn diagram showing the relationships among the sets of shapes in each part below. If an overlap is a particular set of shapes that we have a name for, label it.

a. Trapezoid, rectangle, quadrilaterals with congruent diagonals

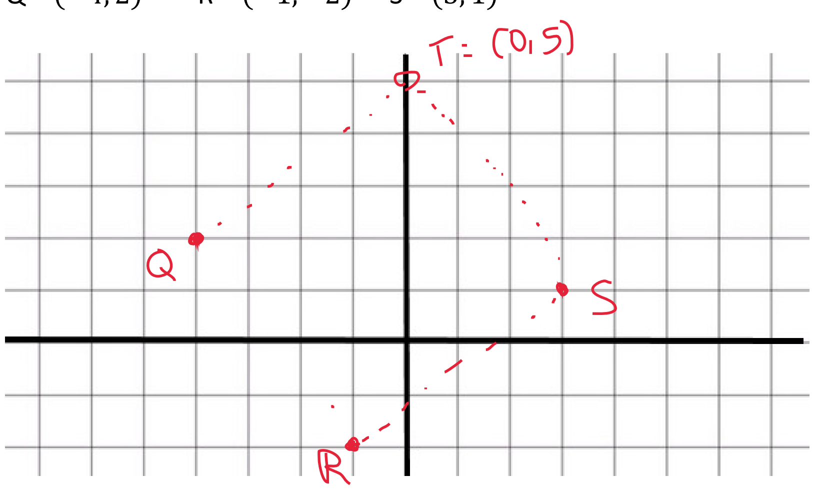


b. Rhombus, parallelogram, kite



5. (12 pts) The following three points, Q, R, and S, are three corners of a quadrilateral. Identify a fourth point, T, so that QRST is a square. Graph QRST on the grid below. Show work to justify that QRST satisfies the definition of a square.

Q = $(-4, 2)$ R = $(-1, -2)$ S = $(3, 1)$



$d = \sqrt{run^2 + rise^2}$

lengths (squares have 4 congruent sides)

Q to R: $\sqrt{(-4-(-1))^2 + (2-(-2))^2} = \sqrt{25} = 5$

Q to T: $\sqrt{(-4-0)^2 + (2-5)^2} = \sqrt{25} = 5$

S to T: $\sqrt{(3-0)^2 + (1-5)^2} = \sqrt{25} = 5$

R to S: $\sqrt{(-1-3)^2 + (-2-1)^2} = \sqrt{25} = 5$

$m = \frac{rise}{run}$

Slopes: (squares have 4 right angles)

Q to R: $\frac{2-(-2)}{-4-(-1)} = -\frac{4}{3}$

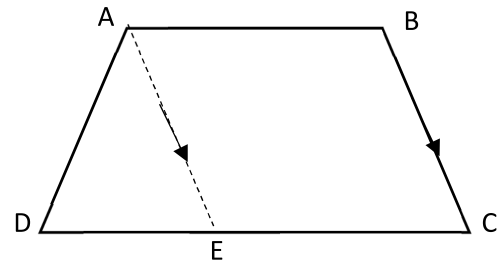
Q to T: $\frac{2-5}{-4-0} = \frac{-3}{-4} = \frac{3}{4}$

S to T: $\frac{1-5}{3-0} = \frac{-4}{3}$

R to S: $\frac{-2-1}{-1-3} = \frac{-3}{-4} = \frac{3}{4}$

negative reciprocals → perpendicular

6. (8 pts) Consider the reasoning in the following proof.



Given: ABCD is an isosceles trapezoid

Prove: <ADE ≅ <BCE

Statement	Reason
1. ABCD is an isosceles trapezoid	1. Given
2. $\overline{AB} \mid\mid \overline{DC}$; $\overline{AD} \cong \overline{BC}$	2. Def'n isosceles trapezoid
3. Draw $\overline{AE}$ so that it goes through point A and is parallel to $\overline{BC}$ ($\overline{AE} \mid\mid \overline{BC}$)	3. Parallel Postulate
4. ABCE is a parallelogram	4. Def'n parallelogram
5. $\overline{AE} \cong \overline{BC}$	5. Opposite sides of a parallelogram are congruent
6. $\overline{AE} \cong \overline{AD}$	6. Substitution
7. <ADE ≅ <AED	7. Base angles of an isosceles triangle are congruent
8. <BCE ≅ <BAE	8. Opposite angles of a parallelogram are congruent
9. <BAE ≅ <ADE	9. Alternate interior angles are congruent when lines are parallel
10. <ADE ≅ <BCE	10. Substitution

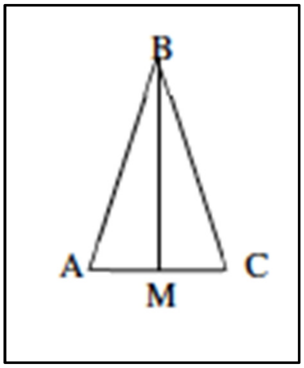
Identify the incorrect step in the proof, explain why it is incorrect.

<BAE and <ADE are not alternate interior angles so 9 is an incorrect step

How should the proof be written in order to correct the error. Clearly indicate which statements and/or reasons need to be replaced, and with what.

"<BAE is conguent to <AED" should replace the statement of step 9, keeping the same reason.

7. (16 pts) Prove the following theorem:

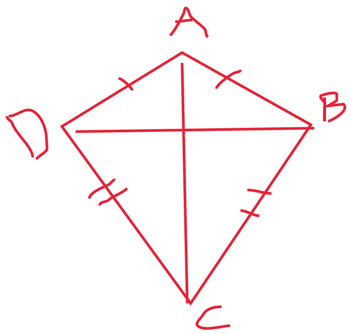


Given: $\overline{AB} \cong \overline{CB}$, $\overline{BM}$ bisects $\angle ABC$	
Prove: $\overline{BM} \perp \overline{AC}$	
1. $\overline{AB} \cong \overline{CB}$ $\overline{BM}$ bisects <ABC	1. Given
2. <ABM ≅ <MBC	2. Def'n of angle bisector
3. $\overline{BM} \cong \overline{BM}$	3. Reflexive property
4. △ ABM ≅ △ MBC	4. SAS
5. <BMA ≅ <BMC	5. CPCTC
6. <BMA+<BMC = 180°	6. Angles that form a straight angle sum to 180.
7. <BMA = <BMC = 90	7. Algebra
8. $\overline{BM} \perp \overline{AC}$	8. Def'n of perpendicular lines.

8. (10 pts) Consider the following conjecture:

In a kite, the diagonals are perpendicular.

a. Sketch an appropriate diagram, label its vertices, and mark the given information on the diagram. Do ***not*** mark the conclusion (the claim that is to be proved).



b. ***Using your labeled diagram***, specify the "Given" and the "To Prove" for this conjecture:

Given: ▱ABCD is a kite with $\overline{AB} \cong \overline{AD}$, $\overline{BC} \cong \overline{DC}$, with diagonals $\overline{AC}$ and $\overline{DB}$.

To Prove: $\overline{AC} \perp \overline{DB}$

YOU DO NOT NEED TO PROVE THIS CONJECTURE!!

Please copy and sign: I pledge on my honor that I have not given or received any unauthorized assistance on this exam. [signed]